

An Efficient Method for Estimating Capability Curve of Virtual Power Plant

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Abstract—To incorporate the operating constraints of a virtual power plant (VPP) in transmission-level operation and market clearing, the concept of the VPP capability curve (VPP-CC) is proposed which explicitly characterizes the allowable range of active and reactive power outputs of a VPP. A two-step projection-based calculation framework is proposed to approximate the VPP-CC by the convex hull of critical points on its perimeter. The output of the proposed algorithm is concise and can be easily incorporated in the existing system operation and market clearing. Case studies based on the IEEE 33 and 123 test feeders show the computational efficiency of the proposed method outperforms existing methods by 4~7 times. Additionally, many fewer inequalities are needed to depict the VPP-CC while achieving the comparative approximation accuracy compared to sampling-based methods, which will relieve the communication and computation burden.

Index Terms—Aggregation model, capability curve, optimal power flow, projection algorithm, virtual power plant.

NOMENCLATURE

A. Indices and Sets

- i, j Distribution system node indices (the root node is indexed 0).
- $\mathcal{L}$ Index set of branches of the distribution system.
- $\mathcal{N}$ Index set of nodes of the distribution system.
- $\mathcal{A}_i$ Index set of nodes adjacent to the node i .

B. Parameters and Constants

- g_{ij}, b_{ij} Conductance and susceptance of the feeder between node i and j .
- $\underline{v}_i, \bar{v}_i$ Lower and upper limits of nodal voltage magnitude.

- $\bar{S}_{ij}$ Transmission capacity of the feeder between node i and j .
- $\bar{S}_0$ Capacity of the substation transformer.
- $\underline{P}_i^g$ Lower bound on active power output of DER at node i .
- $\bar{P}_i^g$ Upper bound on active power output of DER at node i .
- $\underline{Q}_i^g$ Lower bound on reactive power output of DER at node i .
- $\bar{Q}_i^g$ Upper bound on reactive power output of DER at node i .
- $\bar{S}_i^g$ Capacity of DER at node i .
- $\underline{\phi}_i^g, \bar{\phi}_i^g$ Minimum and maximum power-factor angles of DER at node i .
- P_i^d, Q_i^d Active and reactive power demand at node i .

C. Variables

- u_i The square of voltage magnitude of node i .
- θ_i The voltage phase angle of node i .
- P_{ij}^f, Q_{ij}^f Active and reactive power flow on the feeder from node i to node j .
- P_i^g, Q_i^g Active and reactive power output of DER at node i .
- P_0, Q_0 Active and reactive power output of the VPP.

D. Functions

- $\text{conv}(\cdot)$ Convex hull of a set of vectors.
- $\|\cdot\|$ 2-norm of a vector.

I. INTRODUCTION

THE increasing penetration of distributed energy resources (DER) and emerging distribution controls transform conventional ‘passive’ distribution systems to ‘active’ ones [1]. This transformation brings both opportunity and challenge to electric power system operation. On one hand, renewable resources in local systems may benefit low carbon power supply [2] and sustainable development [3]; on the other hand, the relatively small unit capacity, limited visibility and huge amount of DERs may stress operation and management of both distribution and transmission systems.

A virtual power plant (VPP) provides a service-concentrated interface between transmission and distribution systems, thereby facilitating integration of small-scale DERs [4]. The VPP has successfully found its application in real-world projects, e.g., ‘RegModHarz’ project in German [5], FENIX

Manuscript received February 14, 2020; revised April 29, 2020; accepted June 24, 2020. Date of online publication August 19, 2020; date of current version February 28, 2022. This work was supported in part by National Natural Science Foundation of China under Grant No. 51777102, in part by Beijing Natural Science Foundation under Grant No. 3182017, and in part by the Science and Technology Project of State Grid Corporation of China ‘Research on the electricity market mechanism design and application to promote the accommodation of renewable energy’.

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DOI: 10.17775/CSEEJPES.2020.00400

project in Europe [6], and the commercial building-based VPP in Shanghai, China. In existing literature, the commercial behavior and internal optimal operation of the VPP have been studied extensively, see [7], [8] and [9] for examples. Integration of VPPs in transmission system operation is studied in [10]. However, only active power interaction is considered in this research while the detailed system physics of local systems is not well incorporated. In view of DER operating limits and distribution network security constraints, the active (P) and reactive (Q) power outputs of a VPP are constrained [11]. Scheduling a VPP, neglecting its output capability and operating constraints, may lead to inexecutable dispatch commands and security problems, e.g., power imbalance and voltage violation [12]. Distribution level optimization of active and reactive power flow incorporating internal system operating physics is studied in [2], [13]. However, how to aggregate local DERs to participate in transmission-level optimization is not addressed in these studies. To promote participation of DERs while securing a local network, it is of utmost importance to characterize the aggregated operating characteristics of a VPP. Similar with a synchronous generator, we define the VPP capability curve (VPP-CC) as the allowable range of the VPP output that can be met by the internal distribution system. The VPP-CC depicts the physical operating characteristics of VPP in a similar mathematical form a conventional generators and thus, it can integrate the VPP-represented local system into transmission-level optimal scheduling just following the same way as centralized power plants are managed. Incorporating the VPP-CC in transmission-level dispatch and wholesale market clearing will ensure the clearing results are executable for the VPP without violating the operating limits of internal elements.

Due to the coupling of P and Q power flows, however, the VPP-CC is not a simple rectangular or circular region. Worst still, the nonlinearity and nonconvexity of the power flow equations aggravate the challenge of characterizing the VPP-CC, make the shape of the VPP-CC irregular and even nonconvex. Calculation of the VPP-CC is essentially a variable elimination problem with nonlinear constraints, of which a closed-form expression is hardly attainable [11]. Furthermore, a nonconvex representation of the VPP-CC is also incompatible with existing market clearing and system dispatch algorithms, which are usually based on convex programming [10]. Hence, a linear approximation of the VPP-CC is of interest and is more practical in real-world applications. Existing approximation methods can be classified into two categories, namely, the empirical method and the sampling-based method. The former approximates the VPP-CC with some presupposed regular graphics, e.g., ellipse [14] and zonotope [15]. With the presupposed geometry, fewer parameters are needed to depict the VPP-CC and thus, the representation is concise. The accuracy of this kind of method, however, is relatively low due to the hypothesis on the shape of the VPP-CC. Sampling-based methods are studied in [11], [16] and [17] to estimate the VPP-CC. These methods scan the perimeter of the VPP-CC by solving optimal power flow (OPF) problems with different objective functions repeatedly. This kind of method is more precise but has to calculate a large number of sampling

points on the perimeter of the VPP-CC to guarantee accuracy, making the methods computationally inefficient and the approximation result complicated. In general, there is a conflict among approximation accuracy, computational efficiency and representation conciseness in existing methods, which erects obstacles to application of VPPs and the integration of DERs.

This conflict mainly results from incomprehension of the geometry of the VPP-CC. In this regard, this study revisits the definition of the VPP-CC from a projection perspective and proposes a more efficient approximation algorithm. The proposed algorithm consists of two sequential steps. The first step calculates a linear approximation of the VPP-CC based on linearized network configuration and a polytopic projection algorithm. This preliminary approximation is easy to obtain and may provide critical information on the geometrical structure of the real VPP-CC. Then in the second step, the vertices of the preliminary approximation are mapped to some perimeter points of the VPP-CC by solving OPFs, and the final approximation result is the convex hull of these perimeter points. With the aid of preliminary approximation, only a small number of critical points that depict the overall shape of the VPP-CC have to be calculated. Thus, the proposed method is able to obtain a concisely represented and accurate enough approximation with less computational effort. Additionally, this method yields a set of linear inequalities, which can be readily incorporated in existing market and operation schemes.

Major contributions of the paper are summarized as follows:

1) The concept of VPP-CC is defined and modeled systematically based on the geometrical projection. The VPP-CC explicitly characterizes the full range of allowable active and reactive power flexibility that a VPP-represented distribution system can provide without violating internal technical operating constraints.

2) A two-step algorithm is proposed to approximate the VPP-CC by the convex hull of several critical points on its perimeter. The proposed method leverages the geometrical characteristics of the VPP-CC and thus, can approximate the VPP-CC with a more concise representation and higher computation efficiency.

The rest of this paper is organized as follows: Section II introduces the VPP scheduling scheme, derives the static security constraints model, and formulates the VPP-CC model. Section III introduces the proposed VPP-CC approximation method. Section IV exhibits the calculation procedure based on an illustrative example and validates the efficiency and effectiveness of the proposed method based on the IEEE-33 and IEEE-123 test feeders. Finally, conclusions are drawn in Section V.

II. PROBLEM FORMULATION

A. VPP Scheduling Scheme

A VPP that aggregates all the DERs, as well as the distribution network is considered. The VPP studied in this paper is a network-aware one and is operated by the local distribution system operator (DSO). An illustration of the VPP and the information interaction between the DSO and the transmission system operator (TSO) are shown in Fig. 1. Generally, the

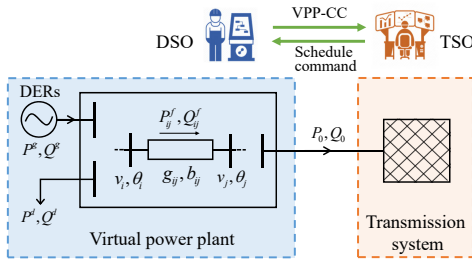


Fig. 1. An illustration of a VPP and the coordination between the TSO and the DSO.

VPP can participate in the transmission-level operation or the wholesale electricity market in the following two-stage scheme [18]:

1) *Scheduling phase*: Before real-time operation, the DSO evaluates and submits the output capability and production offer of the VPP based on the physical and economic parameters of the local system. Then the TSO schedules the P and Q outputs of the VPP incorporating the capability constraints.

2) *Operation phase*: After transmission-level scheduling, the DSO disaggregates the scheduled output to each DERs by running a distribution-level OPF.

This two-stage scheduling scheme enables a set of DERs to contribute to transmission-level balancing and optimization just like a conventional generator. To secure the VPP operation and ensure the scheduled output is executable, it is necessary for the DSO to evaluate the aggregated output capability of the VPP that can be executed in the operation phase, i.e., the VPP-CC.

In consideration of prediction errors of electricity consumption and renewable energy generation, the uncertainty of the VPP capability will increase as the scheduling phase moves more time ahead. In this paper, we focus on the scheduling of VPPs in a short-term basis, e.g., ten-minute or half-hour ahead of operation, which is similar to problem setups in related studies [17], [19]. With this problem setup, uncertainties could be neglected [19]. Since the load and renewable power output in the distribution system are fluctuant, the output capability of the VPP is time-variant and the VPP-CC has to be estimated for each scheduling interval. Thus, a fast and accurate VPP-CC estimating algorithm is necessary to update the VPP-CC timely to support the two-stage scheduling scheme.

B. Security Constraints

The following derivations are for the upcoming single time interval and for simplicity, the time subscript is omitted. For notational conciseness, we assume that each node of the internal network is connected with one DER and one load and then, DERs and loads share the same indices. Assuming three-phase balance, the security constraints of the VPP are formulated as,

$$P_{ij}^f = f_{ij}(\mathbf{v}, \boldsymbol{\theta}), Q_{ij}^f = h_{ij}(\mathbf{v}, \boldsymbol{\theta}), \forall ij \in \mathcal{L} \quad (1)$$

$$P_0^2 + Q_0^2 \leq \bar{S}_0^2, (P_{ij}^f)^2 + (Q_{ij}^f)^2 \leq \bar{S}_{ij}^2, \forall ij \in \mathcal{L} \quad (2)$$

$$P_0 = \sum_{j \in \mathcal{A}_0} P_{j0}, P_i^f - P_i^d = \sum_{j \in \mathcal{A}_i} P_{ij}^f, \forall i \in \mathcal{N} \quad (3)$$

$$Q_0 = \sum_{j \in \mathcal{A}_0} Q_{j0}^f, Q_i^g - Q_i^d = \sum_{j \in \mathcal{A}_i} Q_{ij}^f, \forall i \in \mathcal{N} \quad (4)$$

$$\underline{v}_i \leq v_i \leq \bar{v}_i, \forall i \in \mathcal{N} \quad (5)$$

$$(P_i^g, Q_i^g) \in \mathbf{X}_i, \forall i \in \mathcal{N} \quad (6)$$

Equation (1) represents the alternating current power flows (ACPF) on each branch. Equation (2) enforces the capacity limits of transformer and branches. Equations (3) and (4) refer to active and reactive power balance at each node, and constraints (5) enforce voltage magnitude bounds.

The DER operating limits are represented by (6), where the set $\mathbf{X}_i$ denotes the operating feasible region of the i th DER. The operational limits of DERs depend on their types and specific controlling schemes. In [19], $\mathbf{X}_i$ is modeled by decoupled P and Q bounds. In [20] and [21], capability curves of DERs are modeled by incorporating capacity limits and power factor limits. More elaborate operating models of synchronous machines, wind turbines and PVs are considered in [2], [13], [22]. In this paper, the DER operating constraints in [21] are employed to model $\mathbf{X}_i$, i.e.,

$$\underline{P}_i^g \leq P_i^g \leq \bar{P}_i^g, \underline{Q}_i^g \leq Q_i^g \leq \bar{Q}_i^g \quad (7)$$

$$(P_i^g)^2 + (Q_i^g)^2 \leq (\bar{S}_i^g)^2 \quad (8)$$

$$P_i^g \tan \underline{\phi}_i^g \leq Q_i^g \leq P_i^g \tan \bar{\phi}_i^g \quad (9)$$

Constraints (7)–(9) enforce power output bounds, capacity limit, and power factor limits of the DER, respectively. This model is general enough to depict the operating characteristics of both synchronous generator-based resources and inverter-interfaced resources. Responsive loads can also be incorporated in this model by treating the adjustment amount of the load as a generator. The modeling and parameter of $\mathbf{X}_i$ may influence the shape of the capability curve of the entire VPP, as has been analyzed in [16]. As can be found in Section III, however, the VPP-CC calculation algorithm proposed in this paper does not rely on specific assumptions on the form of the DER operating constraints. This paper mainly focuses on the calculation algorithm of the VPP-CC and thus, the DER operating constraints and their impacts on the shape of the VPP-CC are not discussed in detail in this paper.

C. Linearization of Network Model

To obtain the linear preliminary approximation, the linear network configuration proposed in [23] is employed. This technique models both active and reactive power flows in a linear form and achieves satisfactory accuracy.

Since the preliminary approximation is estimated just to provide information on the geometrical structure of the VPP-CC, the inaccuracy of the network model is acceptable. For power flow equations, regarding $u = v^2$ as independent variable and neglecting network losses, the power flow equations can be approximated in the following linear form,

$$P_{ij}^f = g_{ij} \frac{u_i - u_j}{2} - b_{ij}(\theta_i - \theta_j) \quad (10)$$

$$Q_{ij}^f = -b_{ij} \frac{u_i - u_j}{2} - g_{ij}(\theta_i - \theta_j)$$

Notice the feeder/transformer capacity limits (2) and the DER capacity limits (8) are quadratic and convex, they can

be approximated in the piecewise linear form. Take the capacity limit of the feeder ij for instance, the piecewise linear representation is as follows,

$$\cos \frac{2k\pi}{N} \cdot P_{ij}^f + \sin \frac{2k\pi}{N} \cdot Q_{ij}^f \leq \bar{S}_{ij} \cos \frac{\pi}{N}, \quad k \in [1, N] \quad (11)$$

where N is the number of secant lines that approximate the capacity limit of a branch.

D. VPP-CC Model

In considering the two-stage scheduling scheme of the VPP, the variables in the aforementioned operating constraints can be partitioned into two parts, i.e., 1) coordination variables, denoted by $\mathbf{x} = (P_0, Q_0)$, which are determined in the transmission-level scheduling phase and turn out to be the boundary condition for the VPP in the operation phase; and 2) internal decision and state variables, denoted by $\mathbf{y} = (\mathbf{v}, \boldsymbol{\theta}, \mathbf{P}^g, \mathbf{Q}^g)$ and $\hat{\mathbf{y}} = (\mathbf{u}, \boldsymbol{\theta}, \mathbf{P}^g, \mathbf{Q}^g)$, which are optimized by the DSO in the operation phase to execute the scheduling command from the TSO. Let Ω denote the feasible set of $(\mathbf{x}, \mathbf{y})$ subject to the ACPF-based security constraints (1)-(6). Substituting the feeder capacity limits and power flow equations with (10) and (11), respectively, the feasible set of the linearized security constraints with regard to $(\mathbf{x}, \hat{\mathbf{y}})$ is a polytope, denoted as $\hat{\Omega}$.

The fully modeled VPP-CC can be mathematically defined as the projection of Ω onto the subspace of $\mathbf{x}$, i.e.,

$$\Phi = \{\mathbf{x} \in \mathbb{R}^2 | \exists \mathbf{y}, \text{ s.t. } (\mathbf{x}, \mathbf{y}) \in \Omega\} \quad (12)$$

From the definition, Φ depicts the allowable range of VPP outputs that can be met by at least one internal DER generation portfolio. This definition builds a many-to-one mapping from feasible operating states of the internal system to the allowable P and Q outputs of the VPP, thus characterizing the aggregated flexibility that a set of DERs can provide without violating security operating constraints of the local system. Due to the nonlinearity of the ACPF, the perimeter of Φ is irregular and even nonconvex.

Similarly, the linearized VPP-CC, denoted by $\hat{\Phi}$, is defined as the projection of $\hat{\Omega}$ onto the subspace of $\mathbf{x}$. Because of the limited total generation capacity and load, $\hat{\Phi}$ is a bounded region on the P-Q plane. Based on the linearized network configuration, $\hat{\Omega}$ is a polytope and thus, its projection $\hat{\Phi}$ is a polygon (a two-dimensional polytope) with finite vertices. As a result of neglecting network losses, $\hat{\Phi}$ may overestimate the output capability and underestimate the input capability of the VPP, thus it can not be directly utilized as an approximation of the VPP-CC. However, as will be introduced in the following part, $\hat{\Phi}$ is easy to be calculated and the vertices of $\hat{\Phi}$ provide information that leads to some critical points on the perimeter of Φ . The polygon $\hat{\Phi}$ can act as a guide to obtain an approximation of the VPP-CC and thus, it is referred to as the *preliminary approximation* in this paper.

III. APPROXIMATION METHODOLOGY

In this section, a two-step calculation framework will be introduced that approximates the VPP-CC by the convex hull

of some points on its perimeter. The preliminary approximation $\hat{\Phi}$ is calculated in the first step to provide a guide for identifying the critical points that shape the VPP-CC. Then, the vertices of $\hat{\Phi}$ is mapped to some points on the perimeter of Φ in the second step. The convex hull of the mapped points is the final approximation of the VPP-CC.

A. Step 1: Preliminary approximation

The calculation of $\hat{\Phi}$ is a polytopic projection problem, which can be solved by classic projection algorithms, e.g., the Fourier–Motzkin Elimination (FME) [24]. The FME is a facet-oriented projection method, which calculates the projection by eliminating unconcerned variables in a combined approach. This method is of exponential complexity and may generate a large number of redundant constraints. Note that $\hat{\Phi}$ is two-dimensional and thus, the number of its vertices is not numerous and calculating $\hat{\Phi}$ by enumerating all its vertices may consume fewer computational costs. Hence, the progressive vertex enumeration (PVE) algorithm proposed in [25] is employed to obtain the preliminary approximation. The PVE algorithm outperforms the FME in terms of computational efficiency and avoiding producing redundant constraints when the projected space is low dimensional. For the sake of completeness, the process of calculating $\hat{\Phi}$ by the PVE algorithm is briefly introduced.

The PVE algorithm is summarized in Algorithm 1, which includes an initialization process and a two-layer loop. In the initialization process, vertices of $\hat{\Phi}$ are searched along and opposite each axis. That is, solving the following linear programming with $\boldsymbol{\eta}_j^T = (1, 0), (0, 1), (-1, 0)$ and $(0, -1)$, respectively.

$$h_j = \max_{\mathbf{x}} \boldsymbol{\eta}_j^T \mathbf{x} \text{ s.t. } (\mathbf{x}, \hat{\mathbf{y}}) \in \hat{\Omega} \quad (13)$$

Algorithm 1: Progressive vertex enumeration

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1 initialization:  $\hat{V} = \hat{V}_0, i = 0$ 
2 repeat
3    $i = i + 1$ 
4   convex hull of identified vertices:  $\hat{\Phi}_i = \text{conv}(\hat{V})$ 
5   initialize the set of IR:  $H_i = \emptyset$ 
6   for  $f_j$  on  $\hat{\Phi}_i$  do
7     vertex identification by solving (13):  $\hat{\mathbf{v}}_j, \Delta h_j$ 
8     if  $\Delta h_j > 0$  then
9        $\hat{V}.\text{append}(\hat{\mathbf{v}}_j), H_i.\text{append}(\Delta h_j)$ 
10    end
11  end
12 until  $\max H_i \leq \varepsilon;$ 
13 return  $\hat{\Phi} = \text{conv}(\hat{V})$ 

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Then the initial polygon $\hat{\Phi}_0$ is generated as the convex hull of these vertices, and $\hat{\mathbf{v}}_0$ is calculated as the centroid of the initial polygon.

In the inner loop, (13) is solved with $\boldsymbol{\eta}_j$ equals the outer normal vector of each edge f_j of the current polygon. The geometrical interpretation of this process is moving outward each edge of the current polygon until it reaches an unidentified

vertex of $\hat{\Phi}$. For each newly identified vertex, the improvement ratio (IR) is calculated as follows,

$$\Delta h_j = \frac{h_j^* - \bar{h}_j}{\bar{h}_j - \eta_j^T v_0} \quad (14)$$

where $\bar{h}_j$ is the right-hand side of the equation of the edge f_j . The IR is defined as the relative improvement of the objective of (13) in the inner loop, which quantifies the contribution of the newly identified vertex to the overall shape of $\hat{\Phi}$. Vertices with strictly positive IR will be appended into the worklist $\hat{V}$ and then, the polygon will be updated. This process will be repeated until the maximum IR of the inner loop is smaller than a preset error tolerance ε . As is discussed in Section II-D, $\hat{\Phi}$ is a bounded polygon and the number of its vertices is finite. If ε is set to zero, all the vertices of $\hat{\Phi}$ can be identified, and the accurate $\hat{\Phi}$ can be obtained; otherwise, a positive ε will lead to partial vertices of $\hat{\Phi}$ and consequently, a conservative approximation of the polygon can be obtained. Detailed discussion and proof of the convergence of the PVE algorithm can be found in [25].

B. Step 2: Vertices mapping

In this step, the vertices of $\hat{\Phi}$ obtained in the first step will be mapped to boundary points on the perimeter of the real VPP-CC.

Let $\hat{V}$ denote the set of vertices of $\hat{\Phi}$. For each vertex $\hat{v}_i \in \hat{V}$, let s_i and t_i denote the outer normal vector of its two incident edges. Note that $\hat{v}_i$ is an extreme point of $\hat{\Phi}$; thus, for any c_i satisfying the positive linear combination of s_i and t_i , $\hat{v}_i$ will be the optimal solution of $\max c_i^T \cdot x$ subject to constraints (3)–(6), (10), (11). Solving the problem with the same objective function but the fully modeled AC security constraints, a point v_i on the perimeter of Φ will be obtained. Typically, c_i is set as the average of s_i and t_i , i.e.,

$$c_i = \frac{1}{2}(s_i + t_i) \quad (15)$$

Then, v_i is the optimal solution of the following problem,

$$\begin{aligned} \max & c_i^T \cdot x \\ \text{s.t. } & (x, y) \in \Omega \end{aligned} \quad (16)$$

The above problem is an OPF problem, which can be solved by the primal-dual interior point method [26]. Note that the power flow result of (13) when identifying the vertex $\hat{v}_i$ provides an initial value for solving the OPF problem, which is helpful to obtain a high-quality solution [23]. Map the vertices of $\hat{\Phi}$ to the boundary points of Φ by solving (16). Notice that the mapping of each vertex is independent and thus, parallel computation can be implemented to further accelerate the calculation.

Assuming N_{CP} points are obtained, then the convex hull of all these points is a linearized approximation of the VPP-CC, denoted as Φ_A . Note that Φ_A is a 2-dimensional polygon; thus, it can be represented by exactly N_{CP} linear inequalities.

C. Discussions

The validity of the proposed approximation is based on the following observations on the characteristics of the VPP-CC, 1) Though irregular, the perimeter of Φ is piecewise smooth and consists of segments that are very close to straight lines or arcs. Thus, the overall shape of Φ can be depicted by some critical points on its perimeter, i.e., the junction points of these smooth segments. 2) Based on the linearized network configuration, the overall shape of the preliminary approximation $\hat{\Phi}$ is similar to that of Φ and the vertices of $\hat{\Phi}$ are matched with the critical points of Φ , even though there is a position bias caused by lossless assumption.

The first characteristic is comprehensible since the perimeter of Φ is determined by equality constraints and bounded inequalities of the security constraints, and each segment determined by the same set of bounded inequalities is smooth. The second observation is a natural result of the linearization technique, that the feasible region based on linear approximation is proximate to the accurate one. These two observations will be illustrated through a numerical example in Section IV.

The significance of calculating a preliminary approximation is to reveal the geometrical structure of the VPP-CC and provide a guide for identifying critical points that depict the overall shape of the VPP-CC. Since no specific shape has to be presumed, the proposed method can achieve a more accurate result compared to the empirical method. Compared to the sampling-based method, the proposed method calculates a fewer number of perimeter points to approximate the VPP-CC and thus, may reduce computational costs and lead to a more concise representation of the approximation.

IV. CASE STUDY

In this section, the calculation procedure of the proposed approximation method is first exhibited based on an illustrative example. Then the effectiveness and efficiency of the proposed method are tested based on larger-scale systems.

Optimizations are solved in MATLAB 2017a on a ThinkPad T470 s with an Intel(R) Core(TM) i7-7500U 2.7 GHz CPU. CPLEX 12.7 is utilized to solve linear optimizations, and Matpower Interior Point Solver (MIPS) [26] is chosen as the OPF solver.

A. Benchmark: Sampling-Based Method

The quality and computational efficiency of the proposed method (M_I) is compared with a certain type of the sampling-based method (M_II). This method samples the perimeter of the VPP-CC in a ray-shooting approach, i.e., computing intersection points on the perimeter of the VPP-CC by a set of rays started from an interior point of the VPP-CC.

Mathematically, the sampling method is formulated as follows,

$$\begin{aligned} \max & r \\ \text{s.t. } & x - \hat{v}_0 = (r \cos \alpha, r \sin \alpha)^T \\ & (x, y) \in \Omega. \end{aligned} \quad (17)$$

In (17), the interior point $\hat{v}_0$ is taken as the origin of the polar coordination. α is the polar angle of the ray-shooting

direction. An illustration of this process is exhibited in Fig. 2. Take α evenly in $[0, 2\pi]$ and solve (17) repeatedly, then the perimeter points of the VPP-CC can be obtained. The M_II evenly samples the perimeter of the VPP-CC, thus can be regarded as the ground truth.

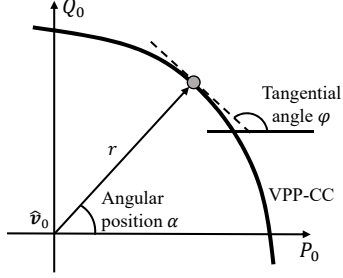


Fig. 2. An illustration of the sampling-based benchmark method.

To validate the effectiveness of the proposed, we also estimate and record the tangential angle φ along the VPP-CC perimeter, as illustrated in Fig. 2. Based on the VPP-CC sampled by M_II, the tangential angle at each point of the perimeter is estimated by the slope of the line determined by two adjacent sampling points. For two adjacent sampling points v_m and v_{m+1} , the polar angle of the concerned perimeter point is the direction angle of the vector $\frac{1}{2}(v_{m+1} + v_m) - \hat{v}_0$; and the estimated tangential angle φ at this perimeter point is the direction angle of the vector $v_{m+1} - v_m$.

B. Numerical Example

1) Description

In this subsection, a numerical example based on a simple two-bus distribution system is constructed to illustrate the calculation procedure of the VPP-CC and to validate the effectiveness of the proposed algorithm. In this test system, a bus with a load and a DER is connected to the root node through one feeder. The active and reactive loads are 3MW and 1MVar, respectively. For the DER, its capacity is 8MVA, with $\underline{P}^g = 0$, $\overline{P}^g = 7\text{MW}$, $\overline{Q}^g = -Q^g = 3\text{MVar}$. The allowable power factor angle of the DER ranges from -60° to 60° . The feeder is rated at 4.5MVA with $r = 1.6\Omega$, $x = 1.6\Omega$. The real VPP-CC is estimated by M_II with 100 sampling points, as shown in Fig. 3.

2) Illustration of the Calculation Procedure

The proposed two-step calculation method is utilized to approximate the VPP-CC. In the first step, the preliminary approximation $\hat{\Phi}$ is calculated by the PVE algorithm. The number of secant lines to linearize each quadratic capability limits is $N = 32$. The error tolerance of the PVE algorithm is $\varepsilon = 0$. The result is plotted as the region with green edges and dot markers in Fig. 3. This polygon has 11 vertices, as the green dot markers display. As a result of neglecting network losses, one may notice that $\hat{\Phi}$ based on the linear network configuration overestimates the output capability and underestimates the input capability of the VPP.

However, it can be seen that the overall shape of $\hat{\Phi}$ is quite similar to the real VPP-CC, and the vertices of $\hat{\Phi}$ can be used as guides to identify critical points on the perimeter of Φ . For

instance, we take the vertex $\hat{v}_1 = (4, 1.13)^T$ to illustrate the vertex mapping procedure in the second step of the calculation framework. The outer normal vectors of the incident edges of the vertex are $s_1 = (0.71, 0.71)^T$ and $t_1 = (1, 0)^T$. Then, the corresponding c_1 equals $\frac{1}{2}[(0.71, 0.71)^T + (1, 0)^T] = (0.86, 0.36)^T$. According to (16), with c_1 as the value vector of the objective in (16), the critical point $v_1 = (3.84, 1.12)^T$ is obtained on the perimeter of the real VPP-CC. Mapping all 11 vertices of $\hat{\Phi}$ to boundary points of the VPP-CC in the same way and constructing the convex hull, the final approximation result is the black edge polygon with square markers shown in Fig. 3. The approximation result is a polygon with 11 vertices, which can be represented by the following 11 inequalities algebraically:

$$\begin{bmatrix} 0.1027 \\ 0.1392 \\ 0.1748 \\ 0.1931 \\ 0.2615 \\ 0.2042 \\ 0.0090 \\ -0.4065 \\ -0.2672 \\ -0.2060 \\ -0.0022 \end{bmatrix} P_0 + \begin{bmatrix} -0.1987 \\ -0.1742 \\ -0.1408 \\ -0.1173 \\ -0.0023 \\ 0.1939 \\ 0.5205 \\ 0.2378 \\ -0.1529 \\ -0.1964 \\ -0.2389 \end{bmatrix} Q_0 \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (18)$$

Though only 11 points on the perimeter are calculated to obtain the approximation, it can be seen that Φ_A is very close to the real VPP-CC, indicating the proposed method is able to achieve acceptable accuracy with relatively concise approximation representation.

3) Effectiveness Validation

Results in Fig. 3 show that preliminary approximation helps to obtain several critical points that determine the overall shape of the VPP-CC and thus leads to a good approximation. The effectiveness of the approximation method is supported by the two observations as introduced in Section III-C. In this part, more analytic results will be exhibited to verify these two observations and to further validate the effectiveness of the proposed calculation framework.

We study the variation of the tangential angle along the VPP-CC perimeter. Based on the 100 sampling points calculated by M_II, the solid blue curve in Fig. 4 exhibits how the tangential angle φ varies along the VPP-CC perimeter. As can be seen, the curve is nearly 'step-wise', which validates the perimeter of the VPP-CC consists of relatively smooth segments. For the segment with polar angle ranges from 295° to 333° , the tangential angle rises, indicating this segment is arc-like and is determined by the bounded feeder capacity limits. For other segments, the tangential angles are almost invariant, indicating these segments are line-like and are determined by voltage limits or by DER output bounds.

The polar angle positions of the identified critical points are also calculated and plotted in Fig. 4. As can be seen, all the turning points of the VPP-CC perimeter are grasped. Additionally, for segments between 295° to 333° , 3 critical points are obtained to approximate the arc-like segments. The proposed calculation method only identifies the junction

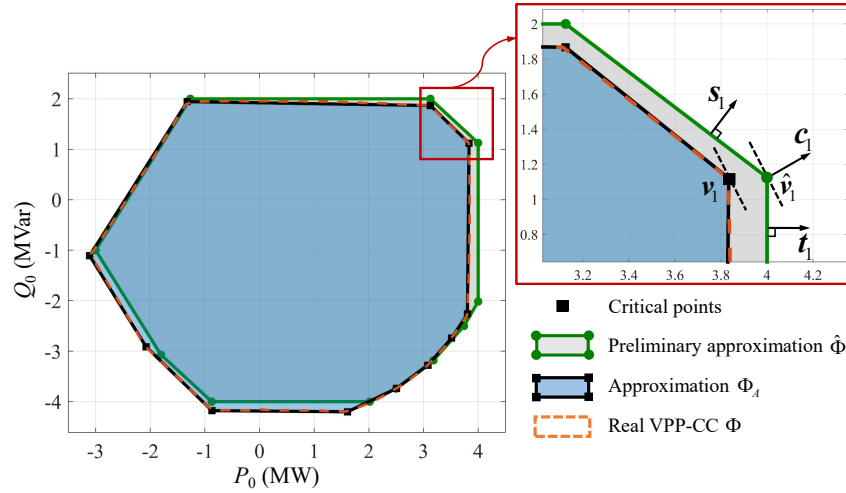


Fig. 3. An illustration of the VPP-CC and the procedure of the proposed approximation method.

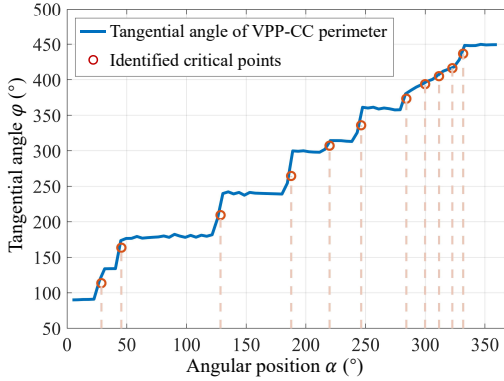


Fig. 4. Tangential angle variation along the VPP-CC perimeter and the position of the identified critical points.

points of line-like segments and the sampling points on arc-like segments, thus it can obtain a high-quality approximation while consuming fewer computational costs.

C. Larger-Scale Cases

1) Description

The efficiency and effectiveness of the proposed method is verified on 4 larger-scale test systems. The test systems are developed based on the IEEE-33 and IEEE-123 test feeders [27]. For each test feeder, VPP implementations with 6 and 12 DERs are tested, respectively. In each testbed, parameters of DERs are identical. Total active and reactive output limits of DERs are set as 70% to the total active and reactive load of the system, respectively, while maximum power factor angles of DERs are 60° . The location of DERs in different test systems are listed in Table I.

The approximation accuracy rate (AAR) is measured by the proportion of the intersection of Φ_A and Φ in their union, formulated as,

$$\text{AAR} = \frac{\sigma(\Phi_A \cap \Phi)}{\sigma(\Phi_A \cup \Phi)} \quad (19)$$

where $\sigma(\cdot)$ represents the area of a set. The ground truth of Φ is estimated by M_II with 100 sampling points. The AAR

equals one if and only if the two sets are identical.

TABLE I
TEST SYSTEM SETUP

#	Feeder	#DER	DER location
C1	IEEE-33	6	3, 6, 9, 12, 22, 26
C2	IEEE-33	12	3, 4, 6, 9, 11, 12, 13, 15, 19, 23, 26, 29
C3	IEEE-123	6	13, 18, 44, 101, 86, 105
C4	IEEE-123	12	1, 13, 18, 25, 26, 35, 47, 76, 89, 102, 105, 108

2) Results

The accuracy, representation complexity and computational efficiency of the proposed method are compared with the benchmark method. The testing results are summarized in Table II. As can be seen, the accuracy of the proposed approximation method is no less than 98.4% in all four cases. A loss of precision is inevitable if a linear approximation is expected. Such errors, however, will not restrict the practicality of the proposed approximation method considering the nominated operating limits of the components are usually conservative and a short-term slight violation is acceptable [28].

TABLE II
PERFORMANCE COMPARISON

Test system	AAR (%)	Inequality number	Computational time (s)	
			M_I	M_II ^a
C1	98.9	14	0.84	5.70
C2	98.6	23	1.49	6.27
C3	98.4	14	1.46	9.15
C4	99.0	24	2.83	11.15

^a With 100 sampling points.

Though there is an approximation error, the representation conciseness and computational efficiency of the proposed method outperforms the benchmark method notably. As shown in Table II, the proposed method requires at most 24 sampling points (and accordingly linear inequalities) to obtain a comparative approximation result as M_II that requires 100 sampling points. The reason for such an improvement is that M_I only identifies the critical points that determine the overall shape of the VPP-CC rather than evenly sampling the VPP-CC perimeter, thus avoiding calculation of points that

contribute less to the shape of the VPP-CC. Fewer sampling points mean a more concise representation of the VPP-CC. Reduction in the approximation result complexity is significant in practice since an intricate representation of the VPP-CC will increase the communication burden and impact the market clearing algorithm when including a large number of VPPs in transmission-level optimizations.

Fewer sampling points also benefit computational efficiency. As can be seen, the computational efficiency of M_I is improved by 4~7 times compared to that of M_{II} . Though the absolute time reduction is not that distinctive in the examined cases, considerable computational costs will be saved for larger-scale cases where solving the OPF becomes more time-consuming. The computational time of M_I is only 2.83 seconds for the 123-node case with 12 DERs, implying the proposed method is quite acceptable in near real-time applications.

In summary, the proposed method can obtain a more concisely represented and accurate enough approximation of the VPP-CC with less computational burden.

V. CONCLUSION

In order to facilitate integration of DERs while ensuring security of the local system, this paper proposes the concept of a VPP capability curve, which explicitly characterizes the allowable range of active and reactive power that a VPP can provide without violating internal operating constraints. A two-step calculation framework is proposed to approximate the VPP-CC. The preliminary approximation is calculated by the PVE algorithm in the first step to reveal the geometrical structure of the real VPP-CC; then the vertices of the preliminary approximation are mapped to some critical points on the perimeter of the VPP-CC to obtain the final approximation. Case studies based on two IEEE standard test feeders with different number of DERs verify the proposed method outperforms the existing method in terms of computational efficiency and representation conciseness.

Taking into account uncertainties and discrete controlled components will be our future work. Estimating the capability curve of hierarchically aggregated VPPs is also worthy of further study.

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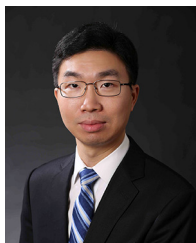
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